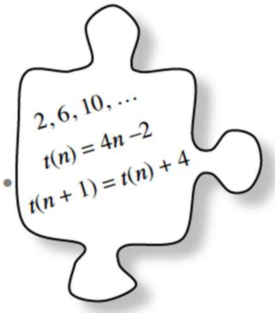
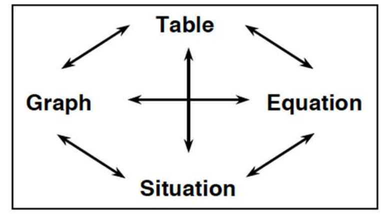
# 0.11 How can I describe a sequence?

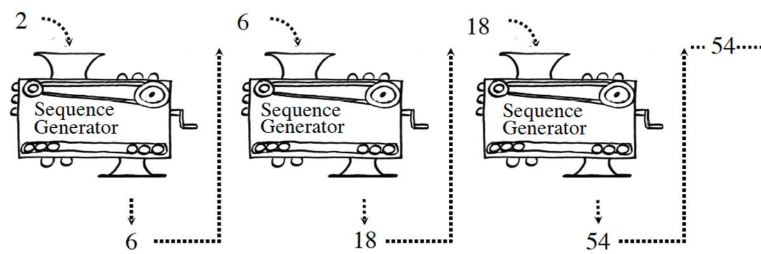



## Generating and Investigating Sequences




In the bouncing ball activity from Lessons 0.9 and 0.10, you used multiple representations (a table, an equation, and a graph) to represent a discrete situation involving a bouncing ball (a situation). Today you will learn about a new way to represent a discrete pattern, called a sequence.

- **1.** Samantha was thinking about George and Lennie and their rabbits. When she listed the number of rabbits George and Lennie could have each month, she ended up with the ordered list below, called a **sequence**. 2, 6, 18, 54, …
- She realized that she could represent this situation using a sequence-generating machine that would generate the number of rabbits each month by doing something to the previous month's number of rabbits. She tested her generator by putting in a **first term** of 2 and she recorded each output before putting it into the next machine. The diagram she used to explain her idea to her teammates is to the right.




   a. What does Samantha's **sequence generator** seem to be doing to each input?
   b. What are the next two terms of Samantha's sequence? Show how you got your answer.
   c. Samantha decided to use the same sequence generator, but this time she started with a first term of 5. What are the next four terms in this new sequence?

- **2.** SEQUENCE FAMILIES
- Samantha and her teacher have been busy creating new sequence generators and the sequences they produce. Below are the sequences Samantha and her teacher created.

| | | | |
|---|---|---|---|
| a. | $-4, -1, 2, 5, \ldots$ | b. | $1.5, 3, 6, 12, \ldots$ |
| c. | $0, 1, 4, 9, \ldots$ | d. | $2, 3.5, 5, 6.5, \ldots$ |
| e. | $1, 1, 2, 3, 5, 8, \ldots$ | f. | $9, 7, 5, 3, \ldots$ |
| g. | $48, 24, 12, \ldots$ | h. | $27, 9, 3, 1, \ldots$ |
| i. | $8, 2, 0, 2, 8, 18, \ldots$ | j. | $\frac{5}{4}, \frac{5}{2}, 5, 10, \ldots$ |

Your teacher will give your team a set of Lesson 0.11 Resource Pages with the above sequences on strips so that everyone in your team can see and work with them in the middle of your workspace.

- **Your Task:** Working together, organize the sequences into families of similar sequences. Your team will need to decide how many families to make, what common features make the sequences a family, and what characteristics make each family different from the others. Read and carry out the directions that follow. As you work, use the following questions to help guide your team's discussion:

How can we describe the pattern? How does it grow? What do they have in common?

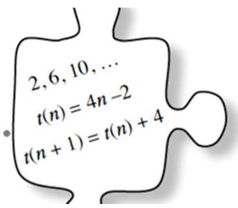
(1) As a team, initially sort the sequence strips into groups based on your first glance at the sequences. Remember that you can sort the sequences into more than two families. You will have a chance to revise your groups of sequences throughout this activity, so just sort them in a way that makes sense to start out with. Which seem to behave similarly? Record your groupings and what they have in common before proceeding.
(2) If one exists, find a sequence generator (growth pattern) for each sequence and write it on the strip. You can express the sequence generator either in symbols or in words. Also record the next three terms in each sequence on the strips. Do your sequence families still make sense? If so, what new information do you have about your sequence families? If not, reorganize the strips and explain how you decided to group them.
(3) Make a table and graph for each sequence. Your table should compare the **term number**, $n$, to the value of each **term**, $t(n)$. This means that your sequence itself is a list of *outputs* of the relationship and the *inputs* are a list of integers! The first term in a sequence is always $n = 1$. Attach each table to the sequence strip it represents. Do your sequence families still make sense? Record any new information or reorganize your sequence families if necessary.
(4) Now graph each sequence. Include as many terms as will fit on the existing set of axes. Be sure to decide whether your graphs should be discrete or continuous. Use color to show the growth between the points on each graph. Attach each graph to the sequence strip it represents. Do your sequence families still make sense? Record any new information and reorganize your sequence families if necessary.

3. Some types of sequences have special names.

a. When the sequence generator *adds* a constant to each previous term, it is called an **arithmetic sequence**. Which of your sequences from problem 2 fall into this family? Should you include the sequence labeled (f) in this family? Why or why not?

b. When the sequence generator *multiplies* a constant times each previous term, it is called a **geometric sequence**. Which of the sequences from problem 2 are geometric? Should sequence (h) be in this group? Why or why not?

# 0.12 How do arithmetic sequences work?




## Generalizing Arithmetic Sequences

In Lesson 0.11, you learned how to identify arithmetic and geometric sequences. Today you will solve problems involving arithmetic sequences. Use the questions below to help your team stay focused and start mathematical conversations. What type of sequence is this? How do we know? How can we find the equation? Is there another way to see it?

- **1.** LEARNING THE LANGUAGE OF SEQUENCES
- Sequences have their own notation and vocabulary that help describe them, such as "term" and "term number." The questions below will help you learn more of this vocabulary and notation.
- Consider the sequence –9, –5, –1, 3, 7, … as you complete parts (a) through (i).
  a. Is this sequence arithmetic, geometric, or neither? How can you tell?
  b. What is the first term of the sequence?
  c. When the sequence generator adds a number to each term, the value that is added is known as the **common difference**. It is the difference between each term and the term before it. What is the sequence generator?
  d. Record the sequence in a table. Remember a sequence table compares the term number, $n$, to the value of each term, $t(n)$.
  e. What is $t(n)$ when $n = 0$?
  f. Graph the sequence. Should the graph be continuous or discrete? Why?
  g. Write an equation (beginning $t(n) =$ ) for the $n^{th}$ term of this sequence.
  h. What is the domain for the sequence equation that you have written?
  i. How is the **common difference** related to the graph and the equation? Why does this make sense?
- **2.** Consider the sequence $t(n) = -4, -1, 2, 5, \ldots$
  a. If the first term is $t(1)$, what is $t(0)$ for this sequence? What is the common difference?
  b. Write an equation for $t(n)$. Verify that your equation works for each of the first 4 terms of the sequence.
  c. Is it possible for $t(n)$ to equal 42? Justify your answer.
  d. For the function $f(x) = 3x - 7$, is it possible for $f(x)$ to equal 42? Explain.
  e. Explain the difference between $t(n)$ and $f(x)$ that makes your answers to parts (b) and (c) different.