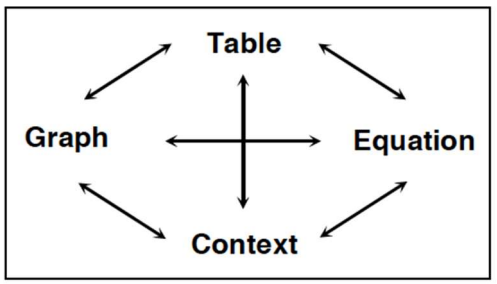
0.5 How can I represent a function?

Modeling a Geometric Relationship

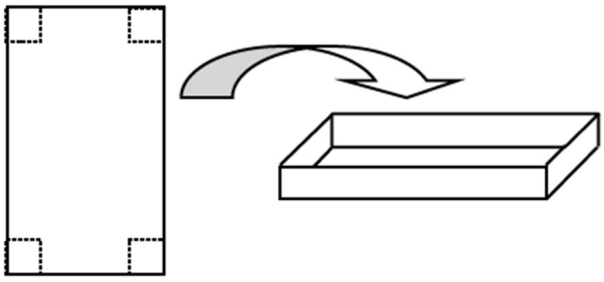
Mathematics can be used to model physical relationships to help us understand them better. Mathematical models can assume the form of a series of diagrams, a situation, a table, an equation, or a graph. In this course, you will be given situations to explore by gathering and interpreting data. You will learn to generalize your information so that you can make predictions about cases that you did not actually test. In this lesson, you will analyze a geometric relationship and look for connections among its multiple representations.



1. ANALYZING DATA FROM A GEOMETRIC RELATIONSHIP

Each team will make paper boxes using the instructions given below. Based on the physical models, your team will represent the relationship between the height of the box and its volume in multiple ways.

If it has not been done already, cut a sheet of centimeter grid paper to match the dimensions that your teacher assigns your team. Then, cut the same size square out of each corner, and fold the sides up to form a shallow box (with no lid) as shown below.



Dimensions

22 cm × 16 cm	18 cm × 10 cm
22 cm × 14 cm	15 cm × 15 cm
20 cm × 15 cm	15 cm × 10 cm
20 cm × 9 cm	12 cm × 9 cm

Your Task: As a team you will investigate the relationship between the height of a box (the **input**) and its volume (the **output**). You can build as many boxes as necessary to establish this relationship. Be sure to build all of your boxes out of paper of the same size. Record your information using multiple representations – including diagrams, a table, and a graph. Also record any thoughts, observations, and/or general statements that come up in your discussion of the problem.

Discussion Points

How can we collect data for this relationship?

How much data is enough?

What are all the possible inputs for our function?

How are the different representations related?

2. GENERALIZING

Now you will generalize your results. Generalizing is an important mathematical process. A common way to generalize is to write an equation using algebra.

a. Draw a diagram of one of your boxes. Since this shape is being used to generalize, you want it to represent a relationship between *any* possible input and its output. Therefore, instead of labeling the height with a number, label the height of this box x.
b. Work with your team to calculate the volume (or y-value) for a height of x. It may help you to remember how you calculated the volume when the height was a number and use the same strategy for your new input of x.

3. LOOKING FOR CONNECTIONS

Put your $x \rightarrow y$ table, graph, and equation in the middle of your workspace. With your team, discuss the questions below.

As you address each question, remember to give reasons when you can. Also, if you make an observation, discuss how that observation relates to your table, graph, and equation.

a. Is the domain of the relationship limited? That is, are there some input values that would not make sense? Why or why not? How can you tell using the graph? The $x \rightarrow y$ table? Using the equation? Using the boxes themselves (or diagrams of the boxes)?
b. Is the range of the relationship limited? That is, what are all of the possible outputs (volumes)? Are there any outputs that would not make sense? Why or why not?
c. Should you connect the points on your graph with a smooth curve? That is, should your graph be *continuous* or *discrete*? Explain.
d. What is different about your graph for this problem when compared to others you have seen in previous courses? What special points or features does it have?
e. Work with your team to find as many other connections as you can among your geometric models, your table, your equation, and your graph. How can you show or explain each connection?