0.6 How can I investigate a function?

Function Investigation

What does it mean to describe a function completely? In this lesson you will graph and investigate a family of functions with equations of the form $f(x) = \frac{1}{x-h}$. As you work with your team, keep the multiple representations of functions in mind.

1. INVESTIGATING A FUNCTION, Part One

   Your team will investigate functions of the form $f(x) = \frac{1}{x-h}$ where h can be any number.

As a team, choose a value for h between −10 and 10. For example, if h = 7, then $f(x) = \frac{1}{x-7}$.

Your Task: On a piece of graph paper, write down the function you get when you use your value for h. Then make an $x \rightarrow y$ table and draw a complete graph of your function. Is there any more information you need to be sure that you can see the entire shape of your graph? Discuss this question with your team and add any new information you think is necessary. Do this investigation by hand. Do not use your graphing calculator to graph the function or make the table.

x	y
	-10
	-5
	-2
	-1
	1
	2
	5
	10

Discussion Points

How can we be sure that our graph is complete?

How can we get output values that are greater than 1 or less than −1?

Make sure to choose a variety of values for x!

What input gets an output of 1, -1, 2, -2, 5, -5, 10, -10?

Stumped? Send your recorder/reporter up to ask the teacher!

In your notebook, completely describe the function and provide multiple representations:

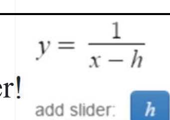
<u>Key Features</u>

- domain and range
- x-int and y-int
- shape of graph
- max and min
- end points or asymptotes

equation	
table	
graph	
key features	

The graph of some functions contains an **asymptote**. To learn more about asymptotes, read the Math Notes box on the next page. Do this now and record any new definitions in your notebook.

2. INVESTIGATING A FUNCTION, Part Two: SUMMARY STATEMENTS



- Go to desmos.com/calculator on your laptop and type $y = \frac{1}{x-h}$. Be sure to add the slider!
- Summary statements are a very important part of this course, so your team will practice making them. A summary statement is a statement about a function along with thorough justification. A strong summary statement should be justified with multiple representations (x → y table, equation, graph, and situation, if applicable). Make as many summary statements about your functions as you can. Remember to justify each summary statement in as many ways as possible.

Function Investigation Questions

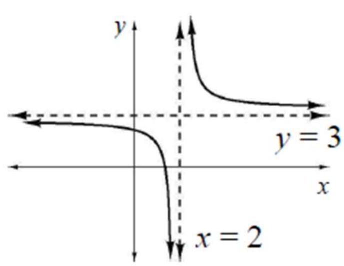
- *What is the shape of the graph?*
- *What happens when x increases?*
- *What happens when x decreases?*
- *When will the graph point upward? Downward?*
- *What are the x- and y-intercepts?*
- *What is the domain of the function? What is the range?*
- *Is there a maximum or minimum y-value?*
- *Does the graph have symmetry?*
- *Are there any important points (like endpoints or a vertex) in this function? Where?*
- *Why are they important?*
- *Does the function have any "problem points" or asymptotes?*
- *Why do they happen?*

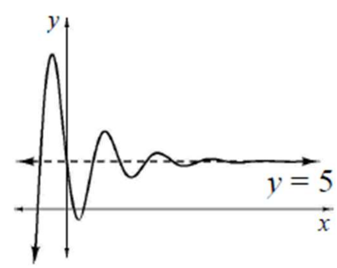
METHODS AND MEANINGS

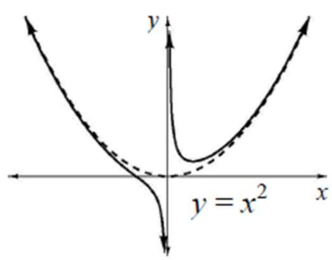
MATH NOTES

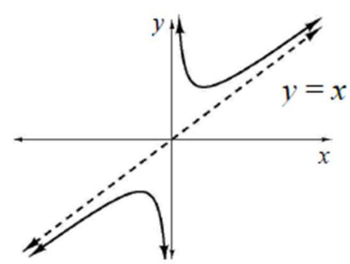
Graphs with Asymptotes

A mathematically clear and complete definition of an asymptote requires some ideas from calculus, but some examples of graphs with **asymptotes** should help you recognize them when they occur. In the following examples, the dotted lines are the asymptotes, and the equations of the asymptotes are given. In the two lower graphs, the y-axis, $x = 0$, is also an asymptote.









As you can see in the examples above, asymptotes can be diagonal lines or even curves. However, in this course, asymptotes will almost always be horizontal or vertical lines. The graph of a function has a **horizontal asymptote** if as you trace along the graph out to the left or right (that is, as you choose x-coordinates farther and farther away from zero, either toward infinity or toward negative infinity), the distance between the graph of the function and the asymptote gets closer to zero.

A graph has a **vertical asymptote** if, as you choose x-coordinates closer and closer to a certain value, from either the left or right (or both), the y-coordinate gets farther away from zero, either toward infinity or toward negative infinity.