1.11 Price Points

Part I – Custom T-Shirts

You've decided to expand your business to create custom t-shirts. When a customer places an order for a special design, you charge a one-time fee of $15 to set up the design plus $8 for each t-shirt printed.

1. Your first responsibility is to make a table that shows how much a customer would be charged for various numbers of shirts. Make a table that includes the cost of 0 to 100 shirts, going up by 25.
2. How much should you charge for 150 shirts? How much should you charge for 750 shirts? Explain how you determined your answers.
3. If you have not done so already, write an equation that could be used to determine how much to charge a customer for any number of shirts.
4. If a customer wanted to know how many shirts they could order for $100, what would you say? Explain how you figured this out.

Part II – Revenue

Your business has now decided to sell a new line of hoodies. A higher selling price per means more money per hoodie but also that as the price increases, the overall sales will decrease. Your marketing team has done research and provided you with some information. The marketing team has given you the following linear function that models the relationship between $\boldsymbol{number\ of\ hoodies\ sold}$ ($\boldsymbol{N}$) and $\boldsymbol{price\ per\ hoodie}$ ($\boldsymbol{x}$) in dollars: $\boldsymbol{N(x) = 2000 - 40x}$

1. What is the y-intercept of this function?
2. Interpret the y-intercept in the context of the problem. What does it mean?
3. What is the x-intercept of this function?
4. Interpret the x-intercept in the context of the problem. What does it mean?
5. If your company were to charge $10 per hoodie, how many would you sell?

Part III – Total Revenue

You were also given the following quadratic function which models the relationship between $total\ revenue\ generated$ (R) and $price\ per\ hoodie$ (x) in dollars. $\mathbf{R}(\boldsymbol{x}) = \boldsymbol{x}(\mathbf{2000} - \mathbf{40}\boldsymbol{x})$

***Note that revenue is found by multiplying the price per hoodie (x) by the number of hoodies sold at that price ($2000 - 40x$) which is the linear function from Part II*

1. According to this function, if you charge $0 per hoodie, how much revenue would your company make? Does that make sense? Explain.
2. According to this function, if you charge $30 per hoodie, how much revenue would your company make? How many hoodies will you sell at that price?
3. Use the provided revenue function to complete the table below in your notebook.

x **Price per hoodie in $**	**R(x)** **Revenue in $**	**2000 – 40x** **# of hoodies sold**
$0		
$10		
$20		
$30	$24,000	800
$40		
$50		

5. At what prices is the revenue equal to $0?
   ***Note that these are also the x-intercepts for $R(x) = x(2000 - 40x)$*
6. Explain what these x-intercepts tell your company in terms of sales, prices, and revenue.

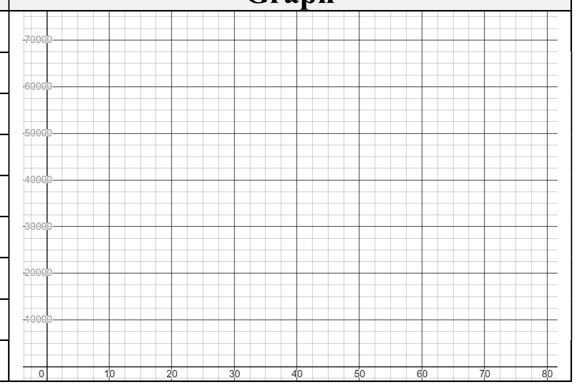
Part IV – Price Point

Your company would like to know what price you should charge to maximize revenue (also called the **price point**). You know that this point would correspond to the vertex of the parabola for this revenue function but are confused because the function was not in vertex form. However, you realize that because of the parabola's symmetry, the vertex should occur midway between the two x-intercepts.

1. What price is halfway between the two x-intercepts?
2. At this price, how many hoodies would be sold?
3. At this price, how much revenue would be generated?
4. According to the revenue function, is this the greatest amount of revenue possible? Explain.

Suppose your company is also selling jerseys and that revenue function is given by: $\mathbf{R(x) = x(4200 - 60x)}$ (where $x\ is\ again\ price\ in\ dollars$ and $R(x)\ is\ total\ revenue$)

5. What are the x-intercepts of this function?
6. Using the method of averaging the x-intercepts, what price is the x-value of the vertex?
7. What revenue does this price generate (y-value of the vertex)?
8. Complete the table below and sketch a graph of this quadratic function in your notebook.

Price per jersey in $	Revenue in $	Graph
$0		
$10		
$20		
$30		
$40		
$50		
$60		
$70		

9. What price should you charge for these jerseys?
10. At this price, how many would you sell?
11. How much revenue would you make?

Summary: When working with quadratic functions and graphing parabolas, what are the advantages to factored form and averaging the x-intercepts method? Explain. How might quadratic functions be useful to businesses?