4.1 Where does the graph go?

Investigating Rational Functions

In your experience with algebra, you have added, subtracted, and multiplied polynomials, but what happens when you divide them? Today you will make some predictions about the graphs that result when two linear functions are combined by adding, subtracting, multiplying, or dividing them.

3-57. COMBINING LINEAR FUNCTIONS INVESTIGATION

Your team will be assigned a pair of linear functions from the list below. Explore using 3-57 Student eTool (Desmos).

$f_1(x) = x - 2$	$f_2(x) = x + 3$	$f_3(x) = x - 4$
$g_1(x) = 2x + 3$	$g_2(x) = 5x - 9$	$g_3(x) = 5x + 8$
$f_4(x) = x - 1$	$f_5(x) = x - 3$	$f_6(x) = x + 4$
$g_4(x) = 2x + 5$	$g_5(x) = 5x - 9$	$g_6(x) = 5x + 6$
$f_7(x) = x - 3$	$f_8(x) = x + 3$	$f_9(x) = x + 2$
$g_7(x) = 2x + 5$	$g_8(x) = 5x + 7$	$g_9(x) = 5x + 3$

Your Task: With your team, find out as much as you can about what happens when you combine the two linear functions using each of the operations of addition, subtraction, multiplication, and division. Use the following steps to guide your investigation.

1. Make your own prediction of the shape of each new graph and draw a quick, rough sketch on your paper.
2. Discuss your prediction with your teammates.
3. Use a graphing calculator to check your team's prediction.
4. Summarize your findings.

Be sure to carefully record all of your work and be prepared to share your summary statements about the results for each operation with the class.

Note: When entering operations into the graphing calculator, you may need to insert extra parentheses so the calculator will follow your intended Order of Operations.

Discussion Points

What does the new graph look like?

What happens when we use the expressions in a different order? Why?

What are the domains and ranges of the new graphs?

3-61. CLOSED SETS

Whole numbers (positive integers and zero) are said to be a closed set under addition: if you add two whole numbers, you always get a whole number. Whole numbers are not a closed set under subtraction: if you subtract two whole numbers, you do not always get a whole number. For example, $2 - 5 = -3$ and -3 is not a whole number.

a. Investigate with your team whether the set of integers is a closed set under addition and under subtraction. Then investigate whether the integers are a closed set under multiplication and under division. Give examples. If you think the set is closed, explain why. If not, give counterexamples.

b. Are single-variable polynomials closed under addition, subtraction, and multiplication? In other words, if you add, subtract, or multiply two polynomials that have the same variable, will you always get a polynomial as your answer? If you think the set is closed, explain why. If not, give counterexamples.