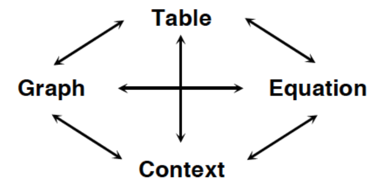
5.1 How can I undo an exponential function?

Finding the Inverse of an Exponential Function

When you first began investigating exponential functions you looked at how their different representations were interconnected, as in the web at right. First semester, you considered how functions and their inverses are related in different representations including equations, $x \rightarrow y$ tables, and graphs. What would the inverse equation for each of the parent functions you worked with first semester look like in each representation?



As you work with your team today, ask each other these questions:
What does the parent function look like in this representation?
How can that help us see the inverse relation?
Would another representation be more helpful?
How can we describe the relationship in words?

- **5-55.** So far, you have learned a lot about eight different parent graphs:

  $y = x^2$ $\quad y = x^3$ $\quad y = x$ $\quad y = |x|$

  $y = \sqrt{x}$ $\quad y = \frac{1}{x}$ $\quad y = b^x$ $\quad x^2 + y^2 = 1$

  a. For each parent, find its inverse, if possible. If you can, write the equation of the inverse in y = form. Include a sketch of each parent graph and its inverse. Remember that you can use the DrawInv function on your graphing calculator to help test your ideas.
  b. Are any parent functions their own inverses? Explain how you know.
  c. Do any parent functions have inverses that are not functions? If so, which ones?

- **5-56.** THE INVERSE EXPONENTIAL FUNCTION
  There are two parent functions, y = |x| and $y = b^x$, that have inverses that you do not yet know how to write in y = form. You will come back to y = |x| later. Since exponential functions are so useful for modeling situations in the world, the inverse of an exponential function is also important. Use $y = 3^x$ as an example. Even though you may not know how to write the inverse of $y = 3^x$ in y = form, you know a lot about it.

. You know how to make an $x \rightarrow y$ table for the inverse of $y = 3^x$. Make the table.

  a. You also know what the graph of the inverse looks like. Sketch the graph.
  b. You also have one way to write the equation based on your algebraic shortcut that you used in part (d) of problem 5-40. Write an equation for the inverse, even though it may not be in y = form.
  c. If the input for the inverse function is 81, what is the output? If you could write an equation for this function in y = form, or as a function $g(x)$ = , and you put in any number for x, how would you describe the outcome?

- **5-57.** AN ANCIENT PUZZLE
Parts (a) through (f) below are similar to a puzzle that is more than 2100 years old. Mathematicians first created the puzzle in ancient India in the 2nd century BC. More recently, about 700 years ago, Muslim mathematicians created the first tables allowing them to find answers to this type of puzzle quickly. Tables similar to them appeared in school math books until recently.

- Here are some clues to help you figure out how the puzzle works:

$\log_2 8 = 3$ $\log_3 27 = 3$

$\log_5 25 = 2$ $\log_{10} 10{,}000 = 4$

- Use the clues to find the missing pieces of the puzzles below:

. $\log_2 16 = ?$

  a. $\log_2 32 = ?$
  b. $\log_? 100 = 2$
  c. $\log_5 ? = 3$
  d. $\log_? 81 = 4$
  e. $\log_{100} 10 = ?$

- **5-58.** How is the Ancient Puzzle related to the problem of the inverse function for $y = 3^x$ in problem 5-56? Show how you can use the idea in the Ancient Puzzle to write an equation in $y =$ form or as $g(x) =$ for the inverse function in problem 5-56.

- **5-59.** THE INVERSE OF ABSOLUTE VALUE

. Find the inverse equation and graph of $y = 2|x + 1|$.

  a. Although you know how to find the table, graph, and equation for the inverse of absolute value, this is another function whose inverse equation cannot easily be written in $y =$ form. In fact, there is no standard notation for the inverse of the absolute value function. With your team, invent a symbol to represent the inverse, and give examples to show how your symbol works. Be sure to explain how your symbol handles that fact that the inverse of $y = |x|$ is not a function or explain why it is difficult to come up with a reasonable notation.