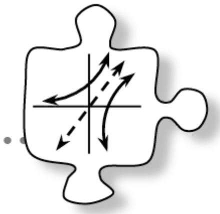
5.2 What is a logarithm?



Defining the Inverse of an Exponential Function

You have learned how to "undo" many different functions. However, the exponential function has posed some difficulty. In this lesson, you will learn more about the inverse exponential function. In particular, you will learn how to write an inverse exponential function in $y =$ form.

- **5-68.** SILENT BOARD GAME
Your teacher will put an $x \rightarrow y$ table on the SMARTBoard that the whole class will work together to complete. The table will be like the one below. See which values you can fill in.

x	8	32	$\frac{1}{2}$	1	16	4	3	64	2	0	0.25	–1	$\sqrt{2}$	0.2	$\frac{1}{8}$
$g(x)$	3		–1												

a. Describe an equation that relates x and $g(x)$.

b. Look back to the Ancient Puzzle in problem 5-57 from yesterday's lesson. If you have not already done so, use the idea of the Ancient Puzzle to write an equation for $g(x)$.

c. Why was it difficult to think of an output for the input of 0 or −1?

d. Find an output for $x = 25$ to the nearest hundredth.

- **5-69.** ANOTHER LOGARITHM TABLE

Lynn was supposed to fill in this table for $g(x) = \log_5 x$. She thought she could use the log button on her calculator, but when she tried to enter 5, 25, and 125, she did not get the ouputs the table below displays. She was fuming over how long it was going to take to guess and check each one when her sister suggested that she did not have to do that for all of them. She could fill in a few more and then use what she knew about exponents to figure out some of the others.

x	$\frac{1}{25}$	$\frac{1}{5}$	$\frac{1}{2}$	1	2	3	4	5	6	7	8	10	25	100	125	625
$g(x)$		–1		0				1					2		3	

a. Discuss with your team which outputs can be filled in without a calculator. Fill those in and explain how you found these entries.

b. With your team, use your calculator to estimate the remaining values of $g(x)$ to the nearest hundredth. Once you have entered several, use your knowledge of exponent rules to see if you can find any shortcuts.

c. What do you notice about the results for $g(x)$ as x increases?

d. Use your table to draw the graph of $y = \log_5 x$. How does your graph compare to the graph of $y = 5^x$?

- **5-70.** Find each of the values below, and then justify your answers by writing the equivalent exponential form.

a. $\log_2(32) = ?$

b. $\log_2(½) = ?$

c. $\log_2(4) = ?$

d. $\log_2(0) = ?$

e. $\log_2(?) = 3$

f. $\log_2(?) = ½$

g. $\log_2(\frac{1}{16}) = ?$

h. $\log_2(?) = 0$

- **5-71.** While the idea behind the Ancient Puzzle is more than 2100 years old, the symbol **log** is more recent. It was created by John Napier, a Scottish mathematician in the 1600's. “Log” is short for **logarithm**, and represents the function that is the **inverse of an exponential function**. You can use this idea to find the inverse equations of each of the following functions. Find the inverses and write your answers in *y* = form.

a. $y = \log_9(x)$

b. $y = 10^x$

c. $y = \log_6(x + 1)$

d. $y = 5^{2x}$

- **5-72.** Practice your logarithm fluency by calculating each of the following, *without changing the expressions to exponential form*. Be ready to explain your thinking.

a. $\log_7 49 =$ _____

b. $\log_3 81 =$ _____

c. $\log_5 5^7 =$ _____

d. $\log_{10} 10^{1.2} =$ _____

e. $\log_2 2^{w+3} =$ _____