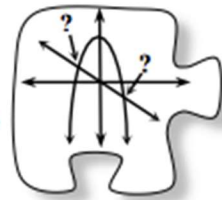


# 3.1 How can I solve?

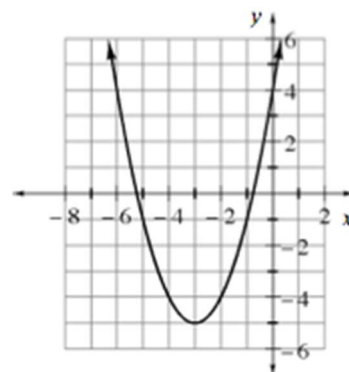
## Strategies for Solving Equations



Today you will have the opportunity to solve challenging equations. As you work with your team, the goal of this section is for you to apply your strategies for solving equations to other types of equations. You will be challenged to use multiple approaches and to write clear explanations to show your understanding.

### • 3-1. SOLVING GRAPHICALLY

- One of the big questions of Units 1 and 2 was how to find special points of a function. For example, you now have the skills to look at an equation of a parabola written in graphing form and name its vertex quickly. But what about the locations of other points on the parabola? Consider the graph of  $y = (x + 3)^2 - 5$  at right. Explore the graph using [3-1 Student eTool](#) (Desmos).



- a. How many solutions does the equation  $y = (x + 3)^2 - 5$  have? How is this shown on the graph?
- b. Use the graph to solve the equation  $(x + 3)^2 - 5 = 4$ . How did the graph help you solve the equation?

### • 3-2. ALGEBRAIC STRATEGIES

The graph in problem 3-1 was useful to solve an equation like  $(x + 3)^2 - 5 = 4$ . But what if you do not have an accurate graph? And what can you do when the solution is not on a grid point or is off your graph?

**Your Task:** Solve the equation below algebraically (that is, using the equation only and without a graph) in at least three different ways. The “Discussion Points” below are provided to help you get started. Be ready to share your strategies with the class.

$$(x + 3)^2 - 5 = 4$$

### *Discussion Points*

What algebraic strategies might be useful?

What makes this equation look challenging? How can we make the equation simpler?

How can we be sure that our strategy helps us find *all* possible solutions?

- **3-3.** Three strategies your class or team may have used in problem 3-2 are **Rewriting** (using algebra to write a new equivalent equation that is easier to solve), **Looking Inside** (reasoning about the value of the expression inside the function or parentheses), and **Undoing** (reversing or doing the opposite of an operation; for example, taking the square root to eliminate squaring). These strategies and others will be useful throughout the rest of this course. Examine how each of these strategies can be used to solve the equation below by completing parts (a) through (f).

$$\frac{x-5}{4} + \frac{2}{5} = \frac{9}{10}$$

- Ernie decided to multiply both sides of the equation by 20 so that his equation becomes  $5(x - 5) + 8 = 18$ . Which strategy did Ernie use? How can you tell?
- Elle took Ernie's equation and decided to subtract 8 from both sides to get  $5(x - 5) = 10$ . Which strategy did Elle use?
- Eric looked at Elle's equation and said, "*I can tell that  $(x - 5)$  must equal 2 because  $5 \cdot 2 = 10$ . Therefore, if  $x - 5 = 2$ , then  $x$  must be 7.*" What strategy did Eric use?
- How many solutions does the function  $y = \frac{x-5}{4} + \frac{2}{5}$  have? How can you use the graph of  $y = \frac{x-5}{4} + \frac{2}{5}$  on your graphing calculator to check your solution to  $\frac{x-5}{4} + \frac{2}{5} = \frac{9}{10}$ ? Where did you look on the graph?
- How can you use the table for  $y = \frac{x-5}{4} + \frac{2}{5}$  on your graphing calculator to check your answer? Where did you look on the table?
- Use the strategies from parts (a) through (c) in a different way to solve  $\frac{x-5}{4} + \frac{2}{5} = \frac{9}{10}$ . Did you get the same result?

- **3-4.** Solve each equation below, if possible, using any strategy. Check with your teammates to see what strategies they chose. Be sure to check your solutions.

a.  $4|8x - 2| = 8$

b.  $3\sqrt{4x - 8} + 9 = 15$

c.  $(x - 3)^2 - 2 = -5$

d.  $(2y - 3)(y - 2) = -12y + 18$

e.  $\frac{5}{x} + \frac{1}{3x} = \frac{4x}{3}$

f.  $|3 - 7x| = -6$

g.  $\frac{6w-1}{5} - 3w = \frac{12w-16}{15}$

h.  $(x + 2)^2 + 4(x + 2) - 5 = 0$

- **3-5.** Some of the solutions from the previous problem can quickly be checked with a graph or table on the graphing calculator. Check the answers for those problems with your graphing calculator.

## 3.2 Solving Radical Equations

For Exercises 1–4, describe each step taken to solve the equation. Then, check the solution to see if it is valid. If it is not a valid solution, explain why.

1.  $\sqrt{x} - 6 = 4$

$$\sqrt{x} = 10$$

$$x = 100$$

2.  $\sqrt[3]{x} - 6 = 4$

$$\sqrt[3]{x} = 10$$

$$x = 1000$$

3.  $\sqrt{x} + 6 = 4$

4.  $\sqrt[3]{x} + 6 = 4$

### Example 1

Solve the following radical equation. Be sure to check your solutions.

$$\sqrt{3x + 5} - 2 = -1$$

### Exercises 5–12

Solve each radical equation. Be sure to check your solutions.

5.  $\sqrt{2x - 3} = 11$

9.  $-12\sqrt{x - 6} = 18$

6.  $\sqrt[3]{6 - x} = -3$

10.  $3\sqrt[3]{x + 2} = 12$

7.  $\sqrt{x + 5} - 9 = -12$

11.  $\sqrt{x^2 - 5} = 2$

8.  $\sqrt{4x - 7} = \sqrt{3x + 9}$

12.  $\sqrt{x^2 + 8x} = 3$

17. Solve the radical equation  $\frac{3}{\sqrt{x+3}} = 1$ . Be sure to check for extraneous solutions.

18. Without solving the radical equation  $\sqrt{x + 5} + 9 = 0$ , how could you tell that it has no real solution?

## 3.3 Solving Radical Equations – Day 2

### Example 1

Solve the equation  $6 = x + \sqrt{x}$ .

### Exercises 1–4

Solve.

1.  $3x = 1 + 2\sqrt{x}$

3.  $\sqrt{x + 5} = x - 1$

2.  $3 = 4\sqrt{x} - x$

4.  $\sqrt{3x + 7} + 2\sqrt{x - 8} = 0$

### Example 2

Solve the equation  $\sqrt{x} + \sqrt{x + 3} = 3$ .

### Exercises 5–6

Solve the following equations.

5.  $\sqrt{x - 3} + \sqrt{x + 5} = 4$

6.  $3 + \sqrt{x} = \sqrt{x + 81}$

#### Lesson Summary

If  $a = b$  and  $n$  is an integer, then  $a^n = b^n$ . However, the converse is not necessarily true. The statement  $a^n = b^n$  does not imply that  $a = b$ . Therefore, it is necessary to check for extraneous solutions when both sides of an equation are raised to an exponent.

# 3.4 Rational Exponents

## Exercise 1

Evaluate each expression.

a.  $\sqrt[4]{81}$

b.  $\sqrt[5]{32}$

c.  $\sqrt[3]{9} \cdot \sqrt[3]{3}$

d.  $\sqrt[4]{25} \cdot \sqrt[4]{100} \cdot \sqrt[4]{4}$

## Exercises 2–12

Rewrite each exponential expression as a radical expression.

2.  $3^{\frac{1}{2}}$

3.  $11^{\frac{1}{5}}$

4.  $\left(\frac{1}{4}\right)^{\frac{1}{5}}$

5.  $6^{\frac{1}{10}}$

Rewrite the following exponential expressions as equivalent radical expressions. If the number is rational, write it without radicals or exponents.

6.  $2^{\frac{3}{2}}$

7.  $4^{\frac{5}{2}}$

8.  $\left(\frac{1}{8}\right)^{\frac{5}{3}}$

9. Show why the following statement is true:

$$2^{-\frac{1}{2}} = \frac{1}{2^{\frac{1}{2}}}$$

Rewrite the following exponential expressions as equivalent radical expressions. If the number is rational, write it without radicals or exponents.

10.  $4^{-\frac{3}{2}}$

11.  $27^{-\frac{2}{3}}$

12.  $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$

## Summary Questions

Write each expression in radical form.

1)  $4^{\frac{1}{3}}$

A)  $(\sqrt[5]{3})^3$

B)  $(\sqrt[3]{3})^4$

C)  $\sqrt[3]{4}$

D)  $(\sqrt[3]{6})^4$

2)  $6^{\frac{4}{3}}$

A)  $(\sqrt[3]{6})^4$

B)  $(\sqrt{7})^3$

C)  $\sqrt[3]{4}$

D)  $(\sqrt[5]{2})^4$

Simplify.

5)  $(100x^6)^{\frac{1}{2}}$

A)  $10x^3$

B)  $x^{16}$

C)  $27x^3$

D)  $\frac{1}{64x^3}$

Write each expression in exponential form.

3)  $\sqrt{x}$

A)  $(10x)^{\frac{3}{4}}$

B)  $x^{\frac{1}{2}}$

C)  $x^4$

D)  $x^{-\frac{3}{2}}$

4)  $(\sqrt[3]{6r})^5$

A)  $(6r)^{\frac{5}{3}}$

B)  $r^{\frac{1}{5}}$

C)  $(3r)^{-\frac{5}{4}}$

D)  $(7r)^{\frac{5}{3}}$

6)  $(81r^2)^{-\frac{1}{2}}$

A)  $512r^9$

B)  $r^9$

C)  $\frac{1}{9r}$

D)  $729r^6$

# 3.5 Properties of Exponents and Radicals

## Opening Exercise

Write each exponential expression as a radical expression, and then use the definition and properties of radicals to write the resulting expression as an integer.

a.  $7^{\frac{1}{2}} \cdot 7^{\frac{1}{2}}$

b.  $3^{\frac{1}{3}} \cdot 3^{\frac{1}{3}} \cdot 3^{\frac{1}{3}}$

c.  $12^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}$

d.  $(64^{\frac{1}{3}})^{\frac{1}{2}}$

## Examples 1–3

Write each expression in the form  $b^{\frac{m}{n}}$  for positive real numbers  $b$  and integers  $m$  and  $n$  with  $n > 0$  by applying the properties of radicals and the definition of  $n^{\text{th}}$  root.

1.  $b^{\frac{1}{4}} \cdot b^{\frac{1}{4}}$

2.  $b^{\frac{1}{3}} \cdot b^{\frac{4}{3}}$

3.  $b^{\frac{1}{5}} \cdot b^{\frac{3}{4}}$

## Exercises 1–4

Write each expression in the form  $b^{\frac{m}{n}}$ . If a numeric expression is a rational number, then write your answer without exponents.

1.  $b^{\frac{2}{3}} \cdot b^{\frac{1}{2}}$

2.  $(b^{-\frac{1}{5}})^{\frac{2}{3}}$

3.  $64^{\frac{1}{3}} \cdot 64^{\frac{3}{2}}$

4.  $(\frac{9^3}{4^2})^{\frac{3}{2}}$

## Example 4

Rewrite the radical expression  $\sqrt{48x^5y^4z^2}$  so that no perfect square factors remain inside the radical.

## Exercise 5

5. Use the definition of rational exponents and properties of exponents to rewrite each expression with rational exponents containing as few fractions as possible. Then, evaluate each resulting expression for  $x = 50$ ,  $y = 12$ , and  $z = 3$ .

a.  $\sqrt{8x^3y^2}$

b.  $\sqrt[3]{54y^7z^2}$

## Exercise 6

6. Order these numbers from smallest to largest. Explain your reasoning.

$16^{2.5}$

$9^{3.6}$

$32^{1.2}$

## 3.6 Graphing Transformations

- You have learned how to move a parabola around a set of axes, write equations, sketch graphs, and model situations. The graph of  $y = x^2$  is called the **parent graph** for the family of parabolas because every other parabola can be seen as a transformation of that one graph. The general equation for the family of parabolas is  $y = a(x - h)^2 + k$ .

This equation tells you how to shift or stretch the parent graph  $y = x^2$ , to get any other parabola. Explore using Desmos.

- Explain what each parameter ( $a$ ,  $h$ , and  $k$ ) represents for the graph of a parabola.
- 1. In this investigation you will use what you have learned about transforming the graph of  $y = x^2$  to transform four other parent graphs. In fact, your team will figure out how to use what you have learned to transform the graph of *any* function!

**Your Task:** As a team, determine how you can make the graph of any function move left, right, up, and down and how you can stretch it vertically, compress it vertically, and flip it. Each team member should investigate one of the following parent functions:  $y = x^3$ ,  $y = \frac{1}{x}$ ,  $y = \sqrt{x}$ , and  $y = |x|$ .

- Remember that to investigate completely, you should sketch graphs, identify the domain and range, and label any important points or asymptotes.
- Then graph and write an equation to demonstrate each transformation you find.
- Finally, find a general equation for your family of graphs. Be sure to use  $a$ ,  $h$ , and  $k$ .

### Discussion Points

How can we move a parabola?

How can we use our ideas about moving parabolas to move other functions?

What changes can we make to the equation?

As a team, place your parent graphs into the middle of your workspace. Make sure everyone copies down a sketch of each parent graph in their notebook. For each graph, identify the domain and range and label any important points or asymptotes.

As a team, write general equations for each given parent equation. Be ready to explain how your general equations work; that is, tell what effect each part has on the orientation (right-side-up or upside-down), relative size (stretched or compressed), horizontal location (left or right shift), and vertical location (up or down shift). Use  $a$ ,  $h$ , and  $k$ .

Next, you will consolidate your knowledge of each of the parent functions that you know and you will identify the importance of the point  $(h, k)$  for each parent function and its family.

- 2. Think about the parent graph for parabolas,  $y = x^2$ .
  - a. Write the equation of a parabola that will be the same as the parent graph, but shifted four units to the right.
  - b. Does the strategy you used to move parabolas horizontally also work for other parent graphs? Justify your answer.
  
- 3. What can the point  $(h, k)$  tell you about how to graph a function from its equation? How can it help you write the equation for a function given its graph? Discuss these questions with your team.
  
- In your Geometry course you transformed figures just like you transformed parent graphs in this course. Next you will look more at geometric transformations and you will explore what happens when you take the opposite of  $x$  before applying the operations of the function. That is, you will investigate  $f(-x)$ .
  
- 3. In Geometry, you called the transformation of figures “translations,” “reflections,” “rotations,” and “dilations.” Refer to problem 2-106 as you complete parts (a) through (d) below. Explore using Desmos.
  - a. What kind of a geometric transformation have you made when you replace  $f(x)$  with  $f(x) + k$ ? Be as specific as you can.
  - b. What kind of geometric transformation occurs when you replace  $f(x)$  with  $-f(x)$ ? Be as specific as you can.
  - c. What kind of transformation is  $f(x - h)$ ?
  - d. What kind of transformation is  $a \cdot f(x)$ ? Be specific.
  
- 4. Investigate the transformation  $y = f(-x)$  as directed below.
  - For each of the parent graphs you have investigated so far, investigate what happens to the graph when you replace  $x$  with  $-x$ . For each parent function, draw the original and the new graph on the same set of axes in different colors. Explore using Desmos.
    - a. For each parent equation, substitute  $-x$  for  $x$  and algebraically simplify the result.
    - b. Describe the geometric transformation that occurs when you replace  $f(x)$  with  $f(-x)$ .
  
- 5. Functions can be categorized as **even functions** or **odd functions**. With your team sort the functions you investigated in problem 2-122 into the following three groups:
  - EVEN FUNCTIONS: All functions where  $f(-x) = f(x)$ .
  - ODD FUNCTIONS: All functions where  $f(-x) = -f(x)$ .
  - FUNCTIONS THAT ARE NEITHER EVEN nor ODD.